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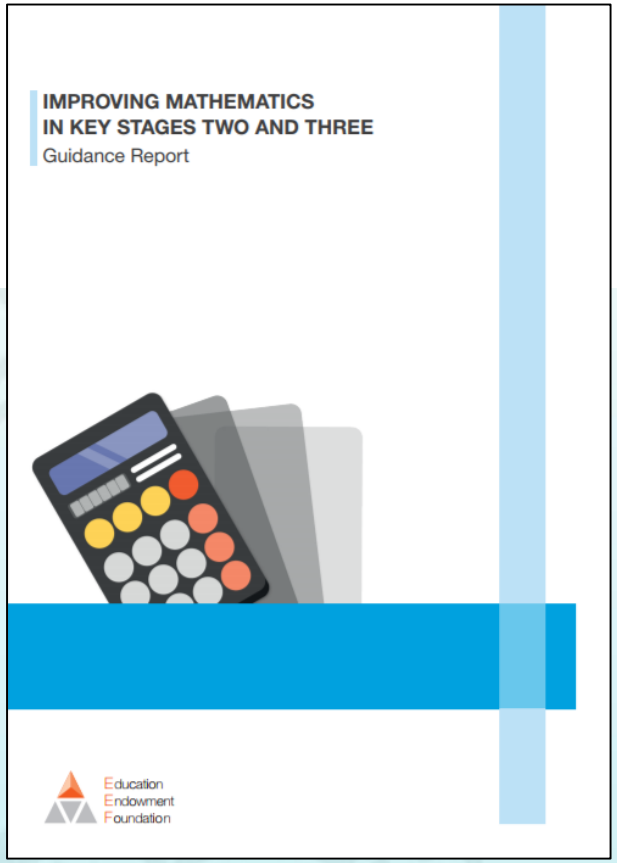
Improving Maths

Delving into the KS2 – KS3 Guidance Report

Tammy Elward
Director of the Derby Research School



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3rd November, 2017

Improving Mathematics in Key Stages 2 and 3

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The Evidence Base

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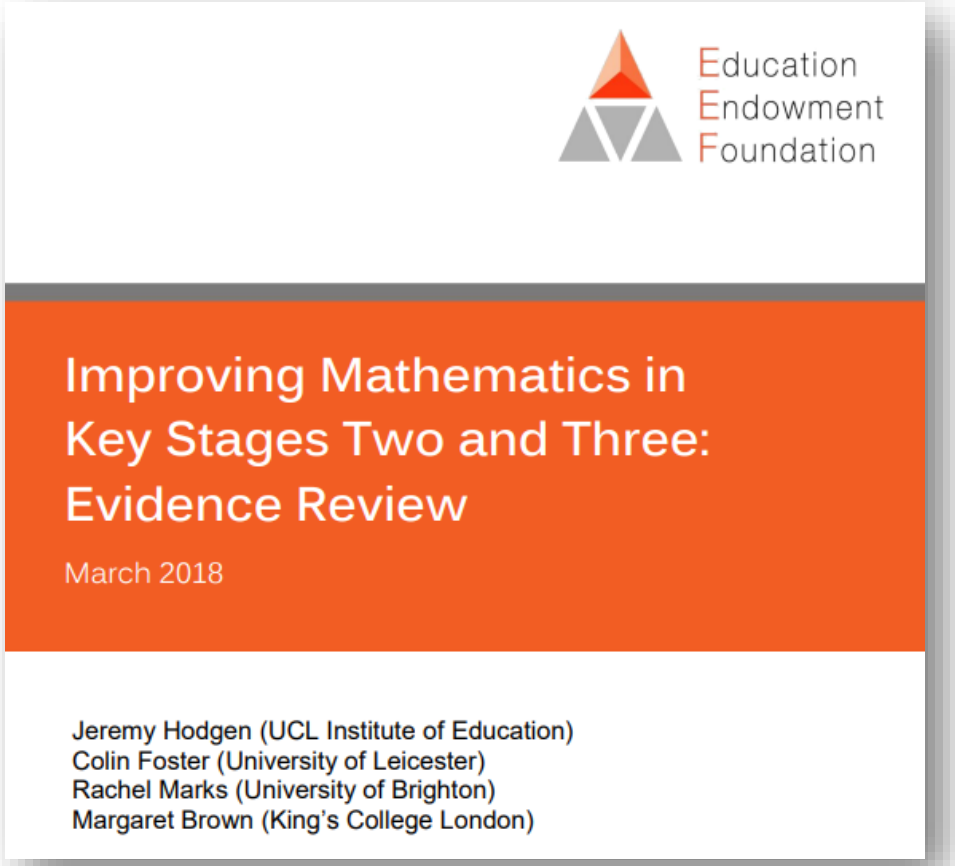
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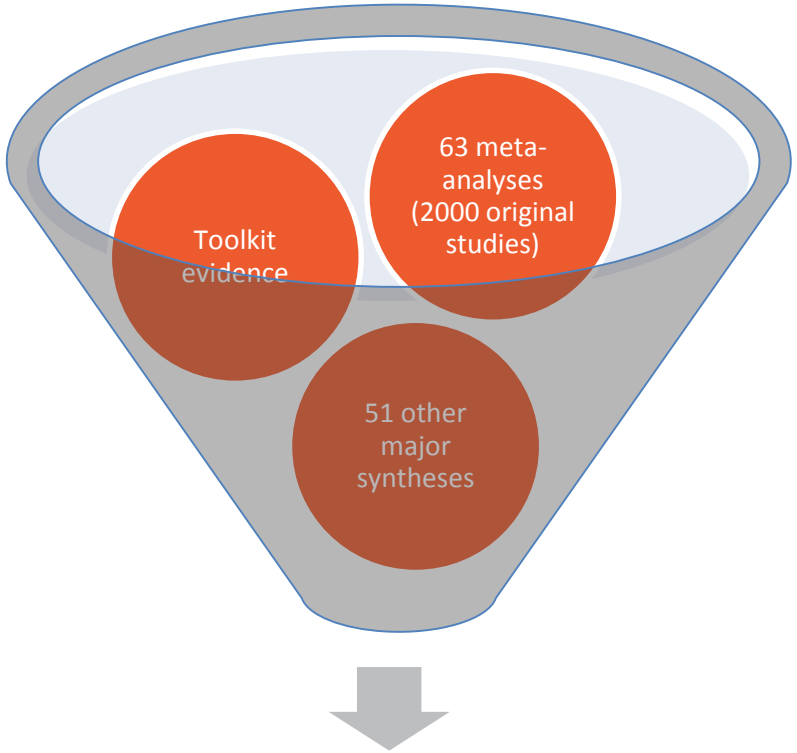
Improving Mathematics in Key Stages Two and Three: Evidence Review

March 2018

Jeremy Hodgen (UCL Institute of Education)
Colin Foster (University of Leicester)
Rachel Marks (University of Brighton)
Margaret Brown (King's College London)



Evidence Review

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Why is evidence important?

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Shared Maths

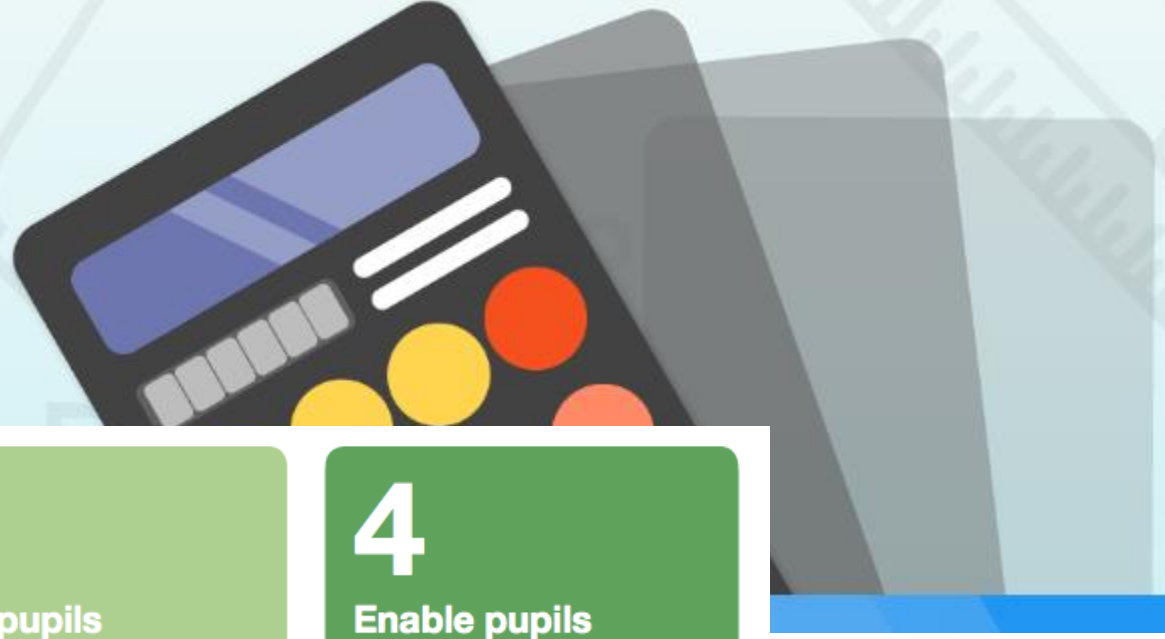
- A cross-age peer tutoring programme. Pupils in year 5 are paired with Year 3 pupils to discuss and work on maths problems.
- Participating pupils spent 20 minutes each week using the approach, for two blocks of 16 weeks over consecutive years.
- Teachers were trained to deliver the intervention.

Catch Up Numeracy

- A one to one intervention for learners who are struggling with numeracy
- It consists of two 15-minute sessions per week, delivered by teaching assistants (TAs).
- The intervention lasted 30 weeks and worked with pupils in years 2-6.

IMPROVING MATHEMATICS IN KEY STAGES TWO AND THREE

Guidance Report



1 Use assessment to build on pupils' existing knowledge and understanding

2 Use manipulatives and representations

3 Teach pupils strategies for solving problems

4 Enable pupils to develop a rich network of mathematical knowledge

5 Develop pupils' independence and motivation

6 Use tasks and resources to challenge and support pupils' mathematics

7 Use structured interventions to provide additional support

8 Support pupils to make a successful transition between primary and secondary school

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Recommendation 2

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2

Use manipulatives and representations

Always a multiple of 9?

Teacher: Give me a two digit number ending in 0.

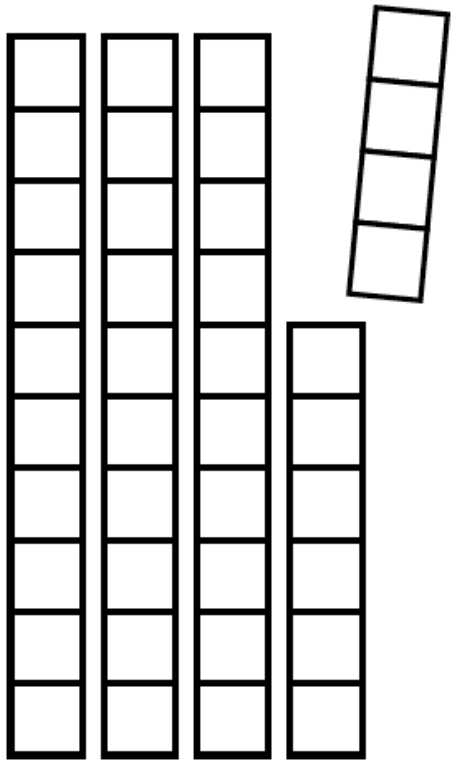
Pupil: Forty.

Teacher: I'm going to subtract the tens digit from the number: 40 – 4 gives me 36.

Try this with other two-digit numbers ending in 0 and the result is always a multiple of 9.

Why does this work?

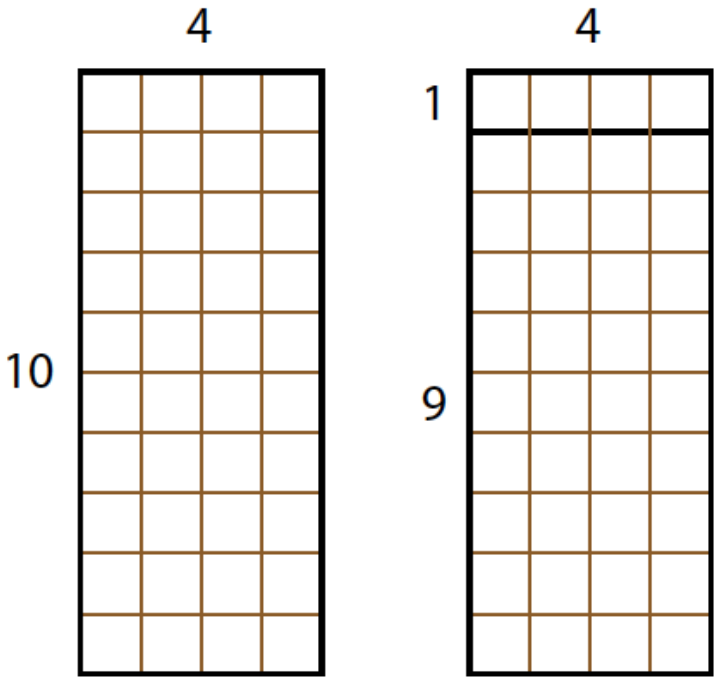
Always a multiple of 9?



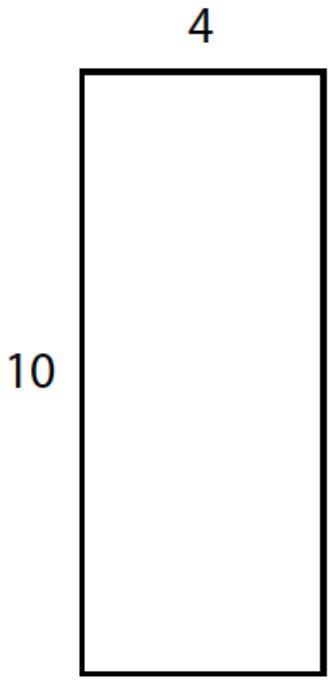
Ah yes! If we take one away from each 10 then we are left with four 9s.

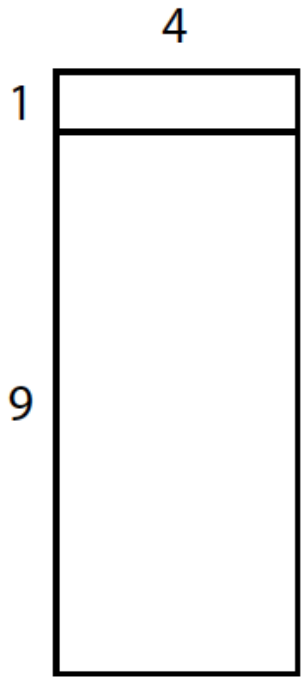
$40 - 4$
$= 10\times4 + 1\times4$
$= (10 - 1)\times4$
$= 9\times4$

Always a multiple of 9?



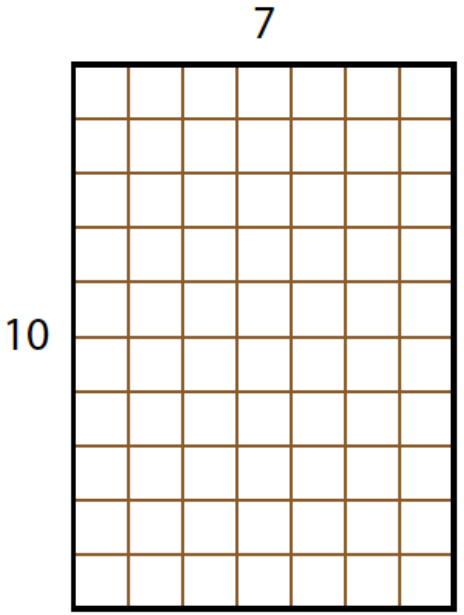
$$40 - 4$$
$$= 10\times4 - 1\times4$$
$$= 9\times4$$

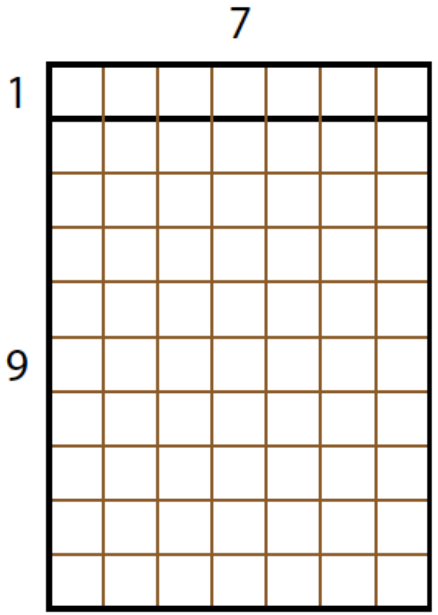




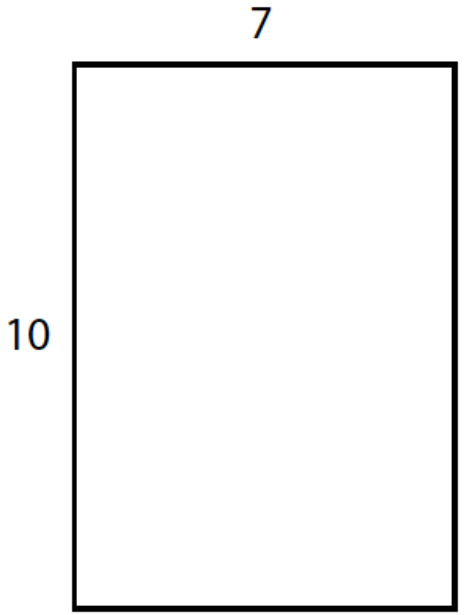
$$40 - 4$$
$$= 10\times4 - 1\times4$$
$$= 9\times4$$

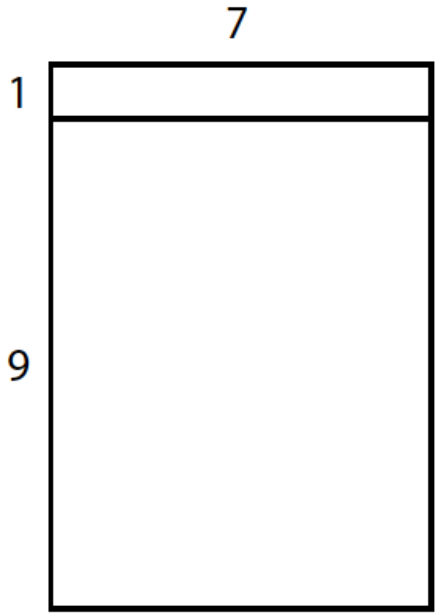
Always a multiple of 9?





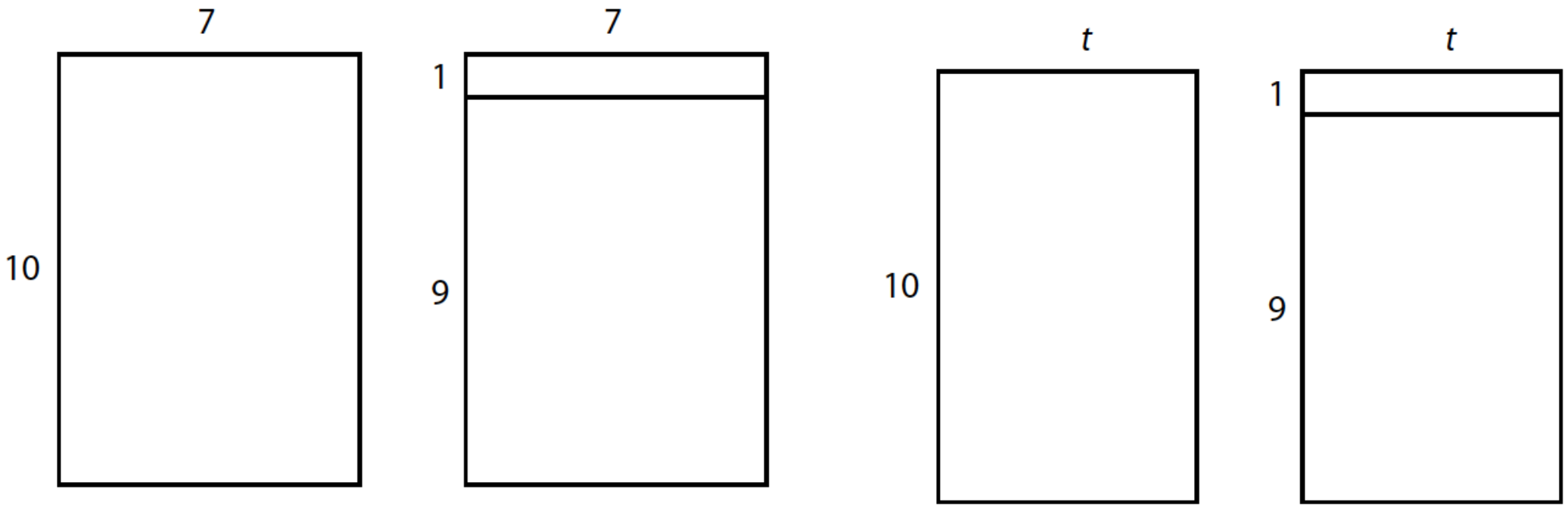
$$70 - 7$$
$$= 10\times7 - 1\times7$$
$$= 9\times7$$





$$70 - 7$$
$$= 10\times7 - 1\times7$$
$$= 9\times7$$

Always a multiple of 9?



$$70 - 7$$
$$= 10\times7 - 1\times7$$
$$= 9\times7$$

$$10t - t$$
$$= (10 - 1)t$$
$$= 9t$$

Is $a^2 + b^2 = (a + b)^2$?

Recommendation 3

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3

Teach pupils strategies for solving problems

3 Teach strategies for solving problems

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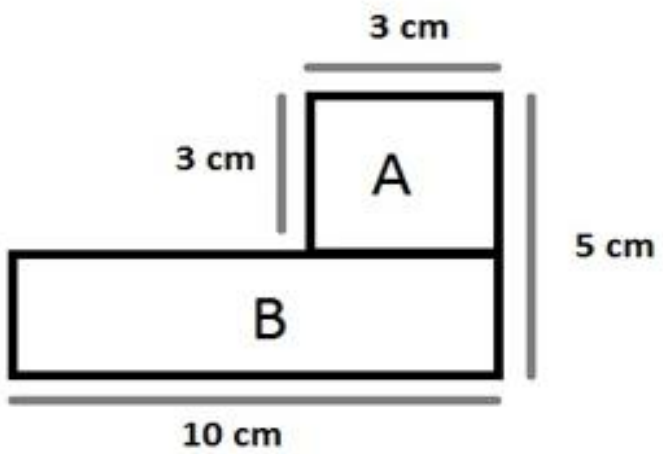
Select genuine problem-solving tasks

Teach pupils to use and compare different strategies

- Draw a picture
- Work backwards
- Identify, and solve, a *simpler* but related problem

Use worked examples

- Promote self-explanation.



Ben and Anna calculated the area of this compound shape in different ways but had the same answer. How is that possible?

10 x 2 = 20
3 x 3 = 9

20 + 9 = 29

Total area is 29cm2

Ben Anna

5 x 10 = 50
3 x 7 = 21

50 – 21 = 29

Total area is 29cm2

3 Teach strategies for solving problems

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Consider and discuss:

Problems with similar structure, but different context

Problems with similar context, but different structure

$12 \div \frac{1}{3}$

Misconception: multiplication makes bigger, division makes smaller

Why does ‘turn over and multiply’ work?

Start with a simpler, but relevant problem.

$$35 \div 5$$

How many 5s in 35?

What do I multiply 5 by to get 35?

$12 \div \frac{1}{3}$

How many ____ in ____?

What do I multiply ____ by to get ____?

$12 \div \frac{1}{3}$

How many ⅓'s in 12?

What do I multiply ⅓ by to get 12?

Recommendation 4

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4

Enable pupils to develop a rich network of mathematical knowledge

4 Enable pupils to develop a rich network of mathematical knowledge

Ensure that pupils develop **fluent recall of facts**

Teach pupils to understand **procedures**

Teach pupils to recognise and use **mathematical structure**

Build on pupils' **multiplicative reasoning** to develop early fraction and division concepts

Teach pupils that **fractions and decimals** extend the number system beyond whole numbers

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When used well, calculators help, not hinder, learning.

2
Use manipulatives and representations

3
Teach pupils strategies for solving problems

4
Enable pupils to develop a rich network of mathematical knowledge

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Study Group Protocol

Working in 3s, take a recommendation each. Take time to read and process the information.

What are the key messages?
What are the important nuances?
What made you reflect the most?

Table 1: Effectiveness of ten learning techniques, from Dunlosky et al (2013) [39]		
High utility	**Practice testing**	Self-testing or taking practice tests on material to be learned.
	Distributed ('spaced') practice	Implementing a schedule of practice that spreads out activities over time.
	Elaborative interrogation	Generating an explanation for why an explicitly stated fact or concept is true.
	Self-explanation	Explaining how new information is related to known information, or explaining steps taken during problem solving.
Moderate utility	**Interleaved practice**	Implementing a schedule of practice that mixes different kinds of problems, or a schedule of study that mixes different kinds of material, within a single study session.
	Summarization	Writing summaries (of various lengths) of to-be-learned texts.
	Highlighting	Marking potentially important portions of to-be-learned materials while reading.
Low utility	**Keyword mnemonic**	Using keywords and mental imagery to associate verbal materials.
	Imagery use for text learning	Attempting to form mental images of text materials while reading or listening.
	Rereading	Restudying text material again after an initial reading.

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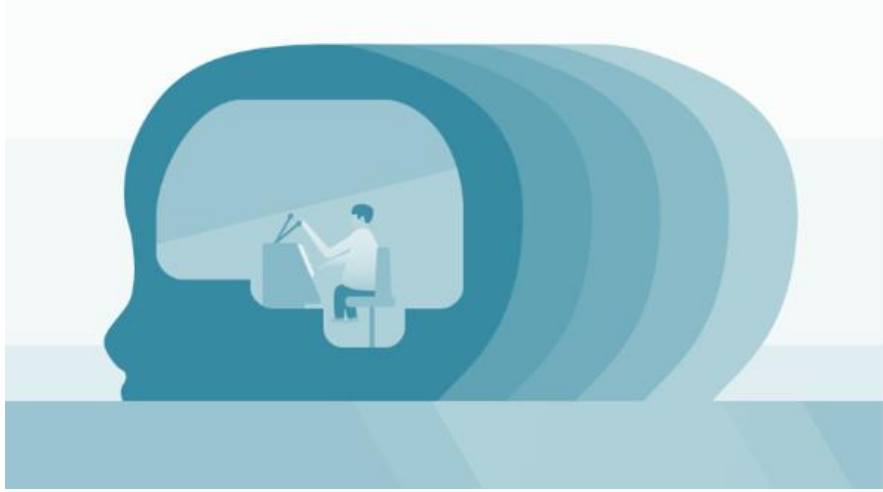

6 Explicitly teach pupils how to organise, and effectively manage, their learning independently

2
Use manipulatives and representations

3
Teach pupils strategies for solving problems

4
Enable pupils to develop a rich network of mathematical knowledge

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Study Group Protocol - JIGSAW

2

Use manipulatives and representations

Our Guide to being a MASTER....

'Explain it!' to me.

'Convince me!' that you are correct.

'Prove it!' to me.

'Use it!' in another problem.

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Explain it!	Convince me!
* Can you explain the problem to me? * What method have you chosen to use, and why have you chosen to use it? * How did you reach your answer?	* How do you know that you are correct? * Can you convince me that it is not this answer?
Use it!	**Prove it!**
* Can you use the same method to solve a different problem? * What prior knowledge can you use to help you solve this problem?	* Can you prove that you are correct using a different method? * Can you represent the same problem in a different way?

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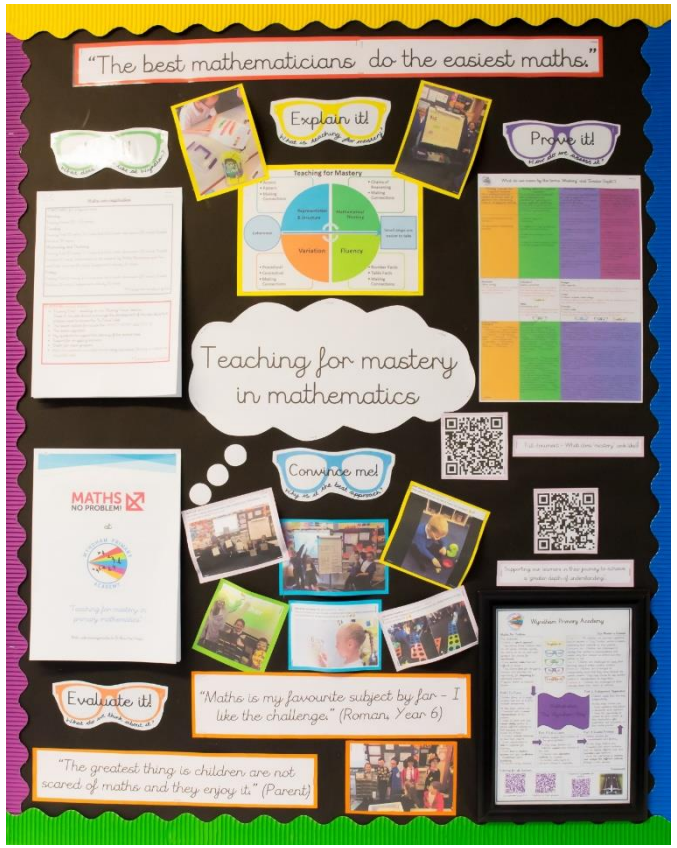
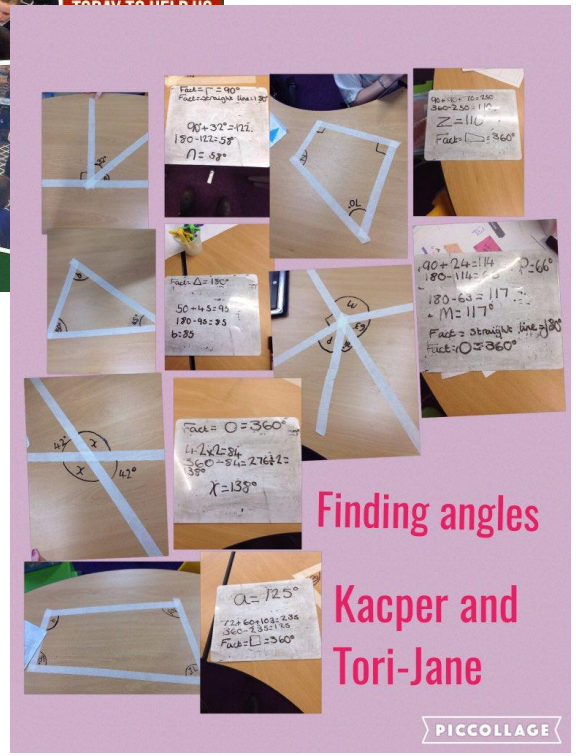
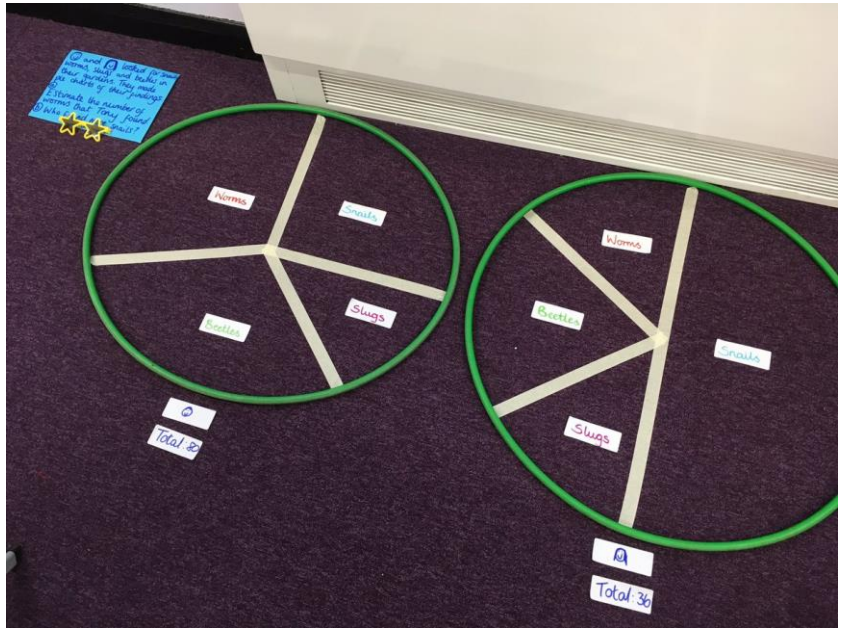
What is Greater Depth/Higher Standard?

Working Towards	Expected with some Mastery	Greater Depth (GD) Higher Standard (HS2)	Greater Depth (GD)+ Higher Standard (HS2)+
This is working towards the expected standard for depth of application and understanding. Pupils do not apply their learning.	This is the expected standard for depth of application and understanding. Pupils show an average application of learning.	This is above the expected standard for depth of application and understanding. Pupils show a good application of learning.	This is well above the expected standard for depth of application and understanding. Pupils show a very good application of learning.
	By the end of the year, children working in this band could be Secure (S) with their age expectations.	By the end of the year, children working in this band could be Secure (S+) with their age expectations.	By the end of the year, children working in this band could be Secure (S++) with their age expectations.
Can remember facts and learnt information. Can explain their understanding with help.	Can use and apply their understanding. Can **explain** how they use facts and information to solve problems. Can use their knowledge in a new context. Can begin to make connections in their knowledge to deepen their learning.	Can use and apply their understanding to more complex problems (e.g. multistep or word problems). Can **explain** how they use facts and information to solve more complex problems. Can make connections in their knowledge to deepen their learning and draw in it in other not just areas to **prove** their claim. What fixed with a mathematician, not **attitude**. Share using their knowledge and understanding. Can reflect on how their understanding helps them to approach problems in different ways.	Can use their understanding to solve more unusual problems, thinking creatively about their strategies to problems. Can **explain** how they use facts and information to solve more complex and unfamiliar problems. Independently can make connections in their knowledge across a range of areas and not just and not **prove** their claim. What fixed with a mathematician, not **attitude**. Share using their knowledge and understanding. Can reflect on how their understanding helps them to approach problems in different ways, **evaluating** how effective and efficient their methods are.
Bloom's Taxonomy **Remember** Define, identify Exhibits previously learned material by recalling facts, terms, basic concepts and answer	Bloom's Taxonomy **Understand** Explain, summarise Demonstrating understanding of facts and ideas by organising, comparing, translating, interpreting, giving descriptions and stating main ideas. **Apply** Demonstrate, show Solving problems by applying acquired knowledge, facts, techniques and rules in a different way	Bloom's Taxonomy **Analyse** Infer, separate Examining and breaking information into parts by identifying motives or causes, making inferences and finding evidence to support generalisations. **Create** Combine, compose, create, design Compiling information together in a different way by combining elements in a new pattern or proposing alternative solutions. **Evaluate** Compare, judge Presenting and defending opinions by making judgments about information, validity of ideas or quality of work based on a set of criteria.	







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3

Teach pupils strategies for solving problems

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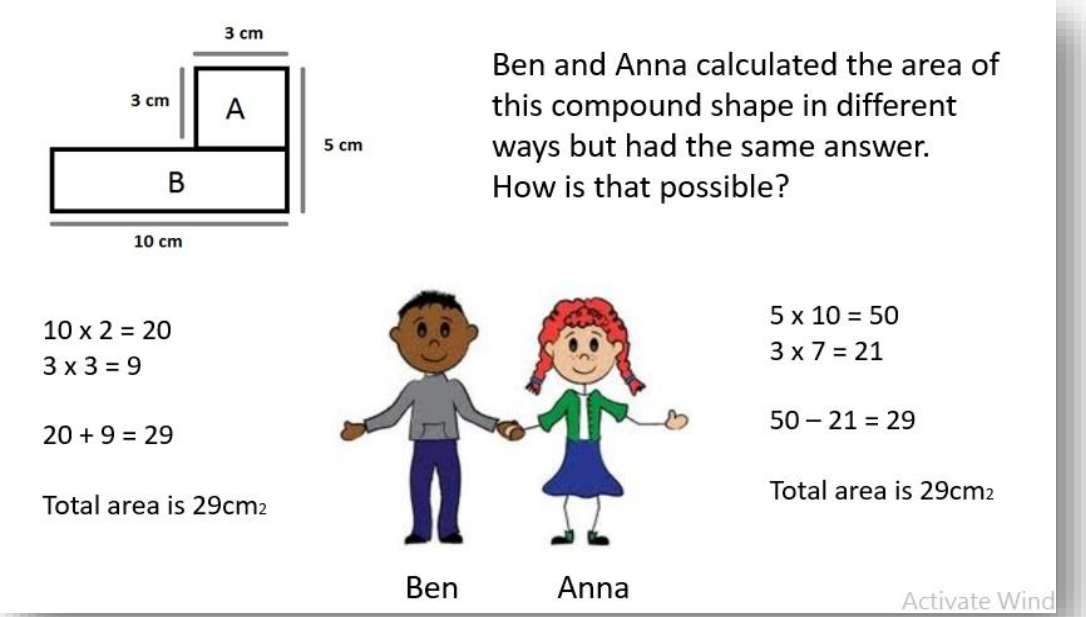
Study Group Protocol - JIGSAW

Share your new learning

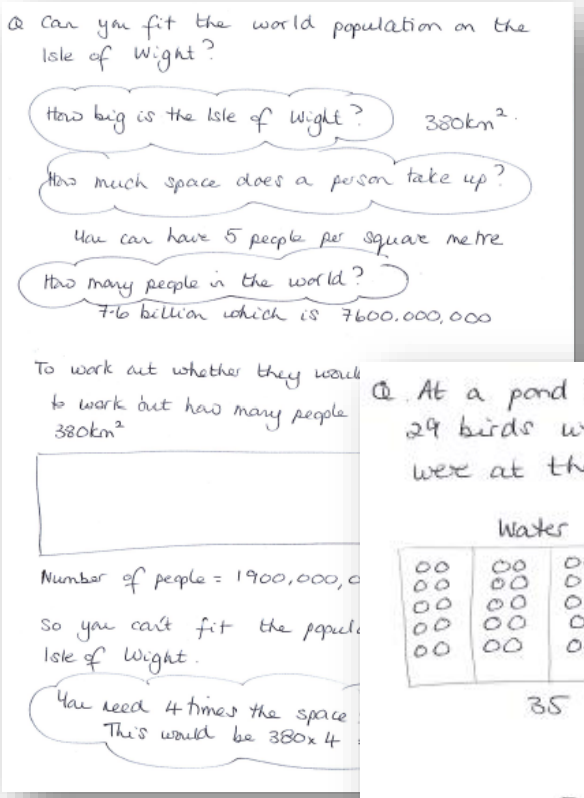
Worked Examples

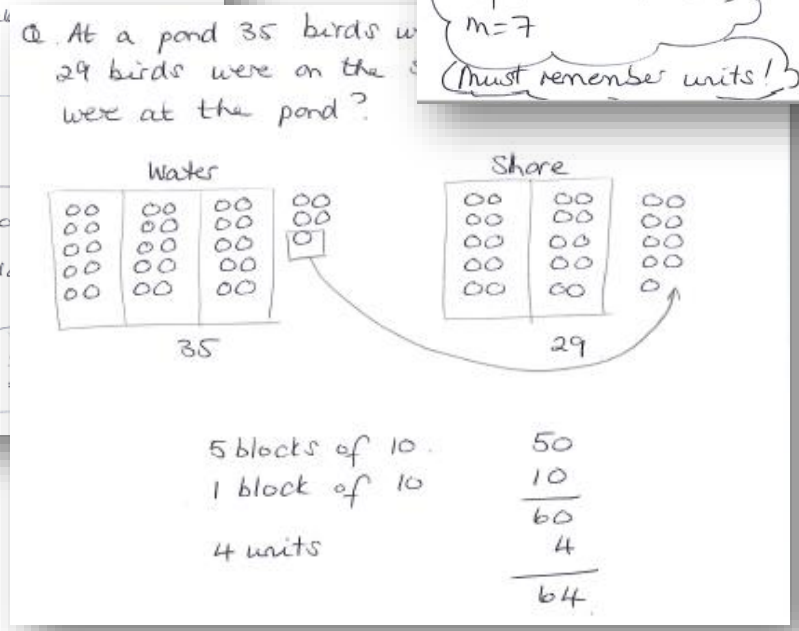
Worked examples, or ‘solved problems’, present the problem and a correct solution together, they remove the need to carry out the procedures required to reach the solution and enable pupils to focus on the reasoning and strategies involved…

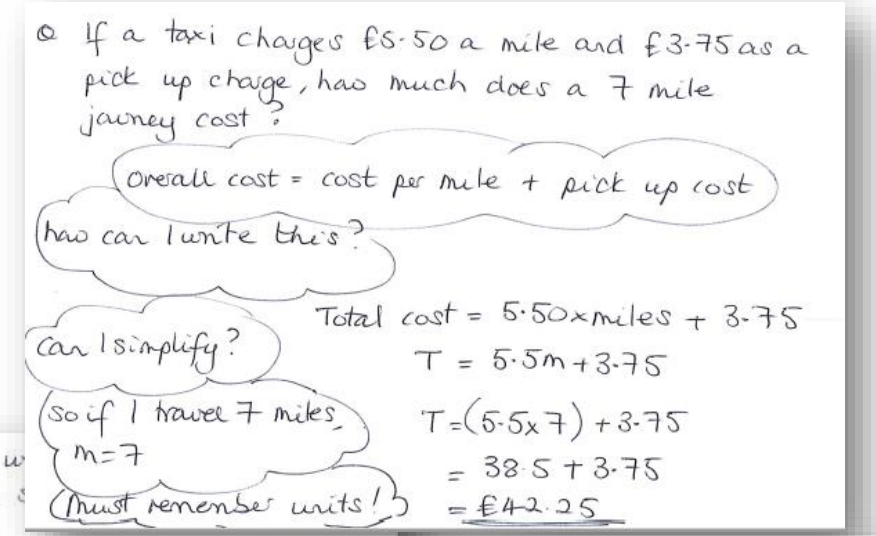
Analysing and discussing worked examples helps students develop a deeper understanding of the logical processes used to solve problems.

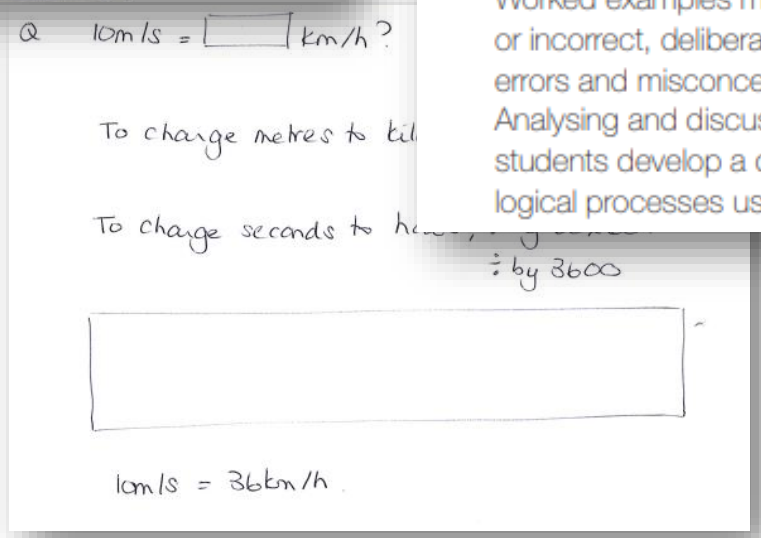


Worked Examples









- **Use worked examples to enable pupils to analyse the use of different strategies.** Worked examples, or 'solved problems', present the problem and a correct solution together, they remove the need to carry out the procedures required to reach the solution and enable pupils to focus on the reasoning and strategies involved. Worked examples may be complete, incomplete, or incorrect, deliberately containing common errors and misconceptions for learners to uncover. Analysing and discussing worked examples helps students develop a deeper understanding of the logical processes used to solve problems.

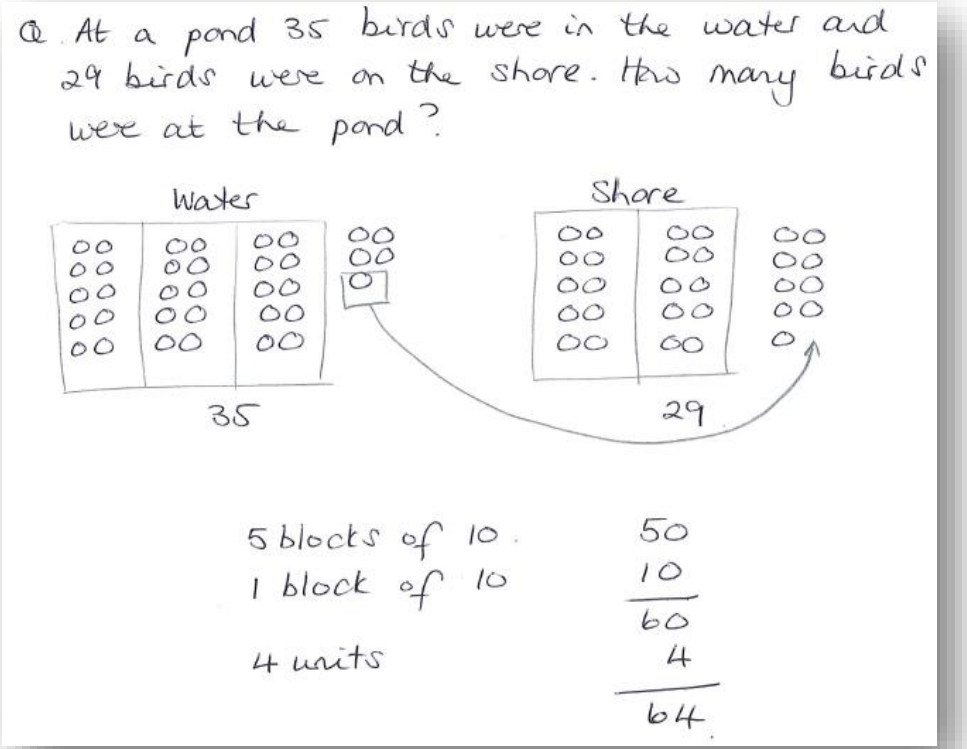
Worked Examples

Look at the worked examples given…

- What approaches have been used? What benefit does it have?

  - Number approach
  - Algebraic approach
  - Explicit thinking is illustrated
  - Misconceptions are included
  - The solution contains missing step(s)

- Do staff have a secure understanding of how to best use worked examples? Do they maximise their impact?



4

Enable pupils to develop a rich network of mathematical knowledge

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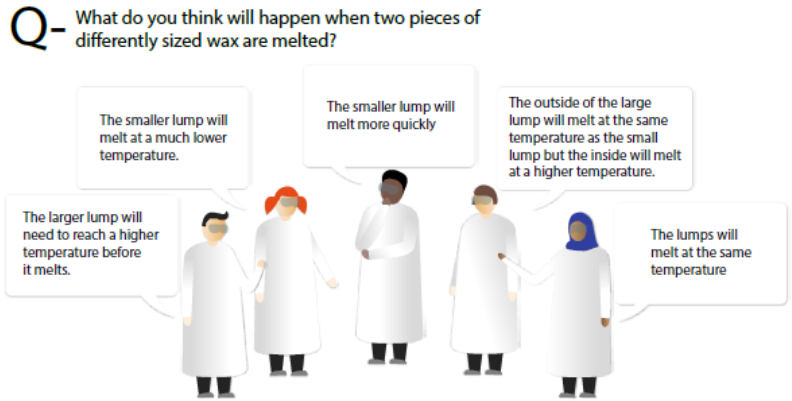
Study Group Protocol - JIGSAW

Share your new learning

Encouraging Exploratory Talk – Talking Heads

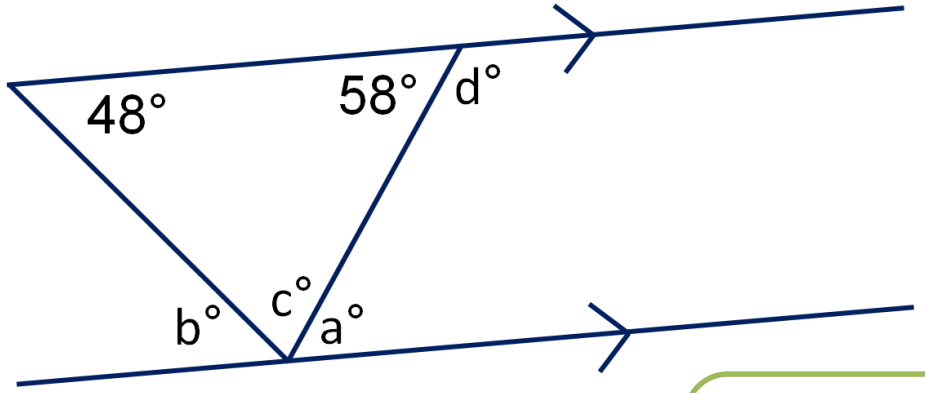
Figure 2: 'Talking heads' to encourage exploratory talk
Adapted from BEST science project[29]



Argumentation is a specific form of dialogue that can help pupils make reasoned claims that are backed by evidence.28 This helps them to understand the power and limitation of scientific knowledge, showing not only what we know but how we know. One way to promote argumentation is to help pupils to move from weaker arguments—which use minimal data and warrants (statements that link data to claims)—to stronger arguments that include greater use of data and rebuttals of counter arguments

Improving Secondary Science – p16 – p17

Encouraging Exploratory Talk – Talking Heads

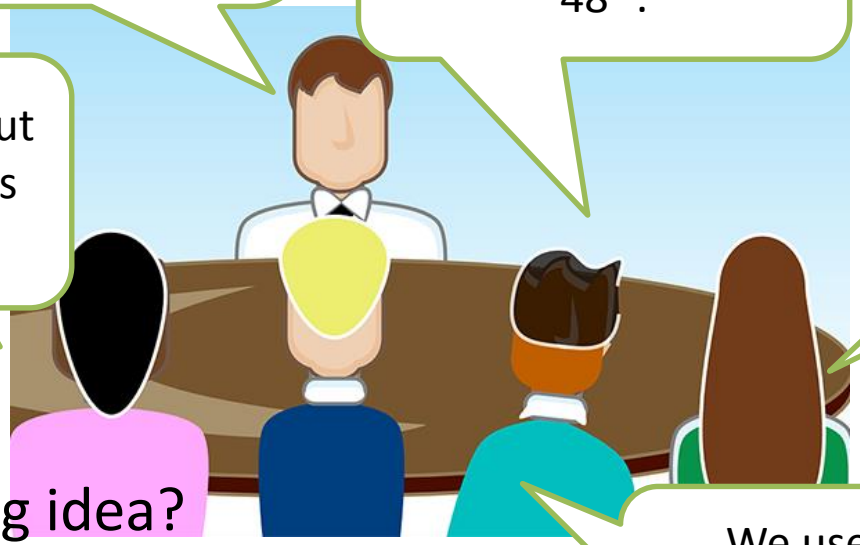


I know angles in a triangle total 180 ° so C has to be 180 ° - (48 ° + 58 °) It's 74 °.

The lines are parallel so A is 58 ° and B is 48 °.

I can see that B + C = D I know D = 180 ° - 58 ° D = 122 ° So B and C are 122 ÷ 2 = 61 °

B is smaller than 90 ° but bigger than 45 ° - it has to be around 50 °



We use a protractor to measure angles so I need to find one.

- Who has the right idea, who has the wrong idea?
- Who gives the best mathematical explanation?
- Who is using evidence, who is expressing an opinion?

‘Quick retrieval of number facts is important for success in mathematics.’

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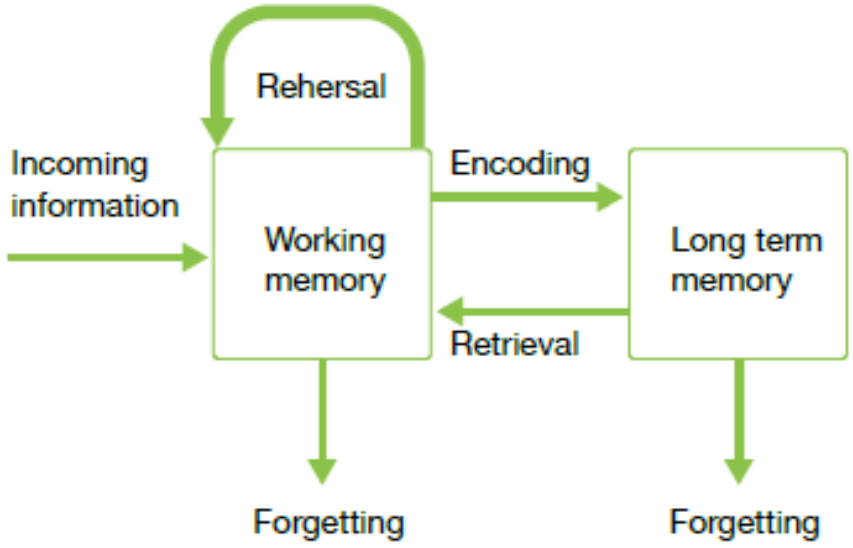
Figure 6: Working memory and long term memory



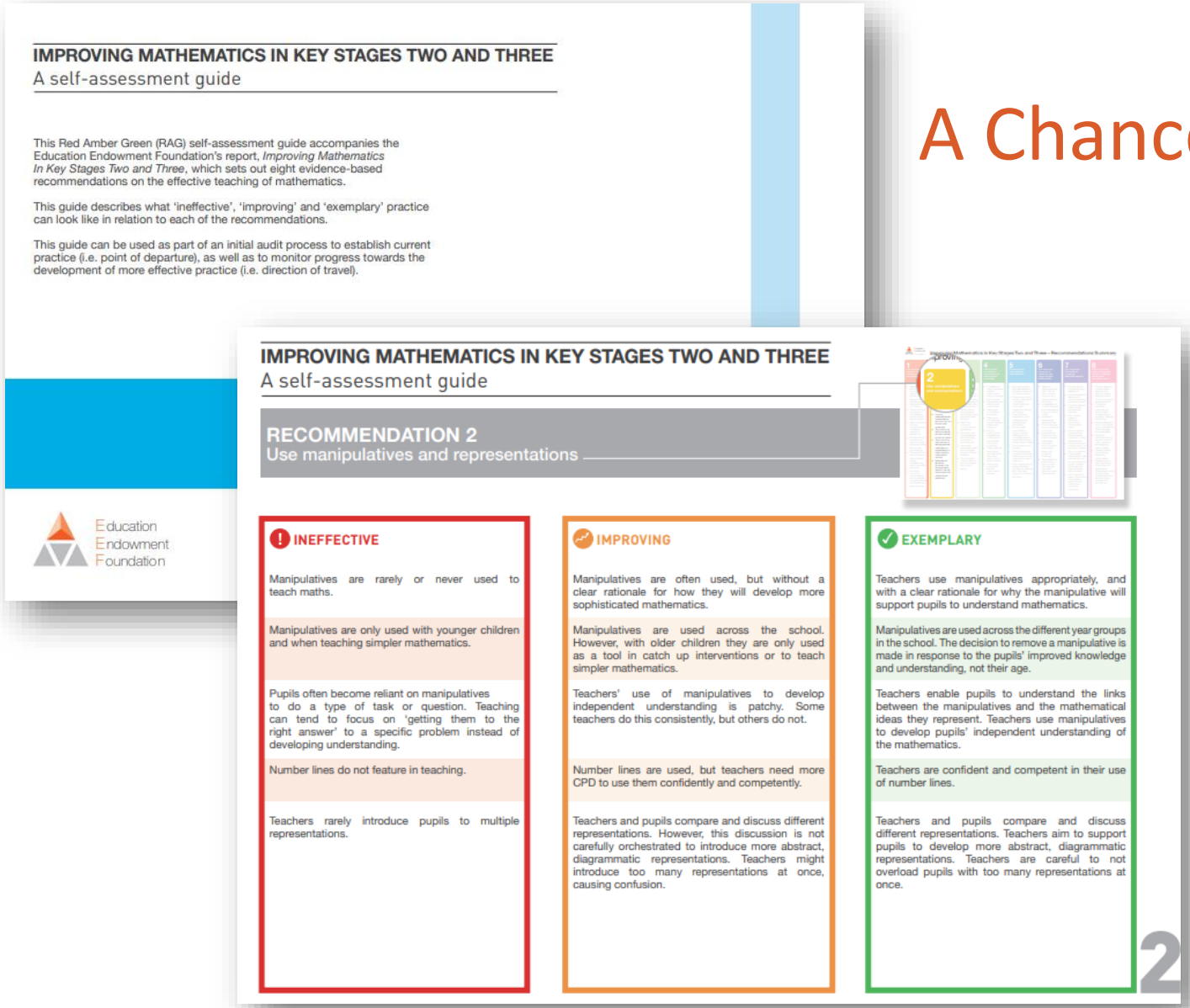
Mathematics programmes of study:

Aims

The national curriculum for mathematics aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

A Chance to Reflect



IMPROVING MATHEMATICS IN KEY STAGES TWO AND THREE
A self-assessment guide

This Red Amber Green (RAG) self-assessment guide accompanies the Education Endowment Foundation's report, *Improving Mathematics In Key Stages Two and Three*, which sets out eight evidence-based recommendations on the effective teaching of mathematics.

This guide describes what 'ineffective', 'improving' and 'exemplary' practice can look like in relation to each of the recommendations.

This guide can be used as part of an initial audit process to establish current practice (i.e. point of departure), as well as to monitor progress towards the development of more effective practice (i.e. direction of travel).

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IMPROVING MATHEMATICS IN KEY STAGES TWO AND THREE
A self-assessment guide

RECOMMENDATION 2
Use manipulatives and representations

INEFFECTIVE	IMPROVING	EXEMPLARY
Manipulatives are rarely or never used to teach maths.	Manipulatives are often used, but without a clear rationale for how they will develop more sophisticated mathematics.	Teachers use manipulatives appropriately, and with a clear rationale for why the manipulative will support pupils to understand mathematics.
Manipulatives are only used with younger children and when teaching simpler mathematics.	Manipulatives are used across the school. However, with older children they are only used as a tool in catch up interventions or to teach simpler mathematics.	Manipulatives are used across the different year groups in the school. The decision to remove a manipulative is made in response to the pupils' improved knowledge and understanding, not their age.
Pupils often become reliant on manipulatives to do a type of task or question. Teaching can tend to focus on 'getting them to the right answer' to a specific problem instead of developing understanding.	Teachers' use of manipulatives to develop independent understanding is patchy. Some teachers do this consistently, but others do not.	Teachers enable pupils to understand the links between the manipulatives and the mathematical ideas they represent. Teachers use manipulatives to develop pupils' independent understanding of the mathematics.
Number lines do not feature in teaching.	Number lines are used, but teachers need more CPD to use them confidently and competently.	Teachers are confident and competent in their use of number lines.
Teachers rarely introduce pupils to multiple representations.	Teachers and pupils compare and discuss different representations. However, this discussion is not carefully orchestrated to introduce more abstract, diagrammatic representations. Teachers might introduce too many representations at once, causing confusion.	Teachers and pupils compare and discuss different representations. Teachers aim to support pupils to develop more abstract, diagrammatic representations. Teachers are careful to not overload pupils with too many representations at once.

2

RAG Tool

- Individual self-reflection
- Phase reflection
- Leadership team

Don't forget to brainstorm the opportunities – collective ideas on how we can improve.

Leader Hat – How can I translate this back to my school?

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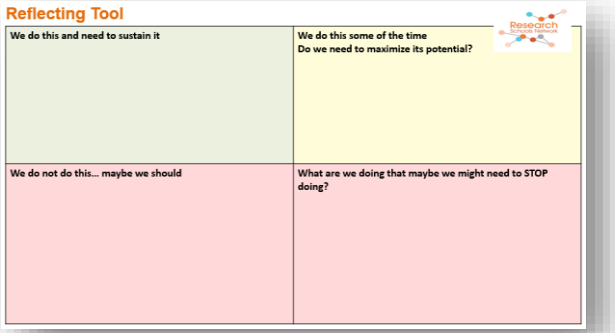
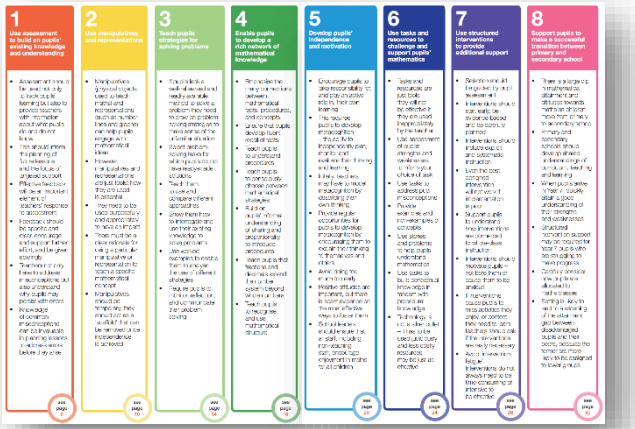
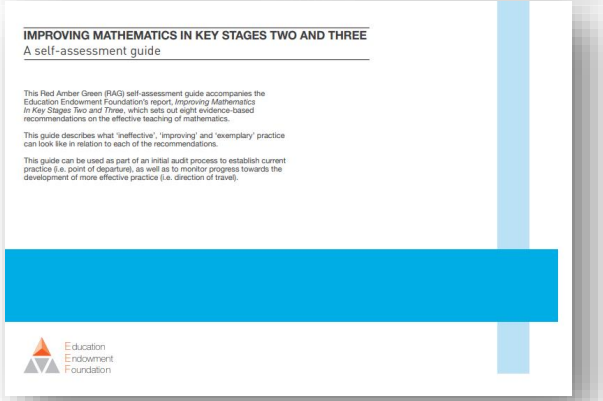
Quick Win Strategies

- ✓ Explore the Guidance Report
  - ✓ Think Piece (all reading same passage)
  - ✓ Study Group (read and share 2/3 strands)
  - ✓ Reflection Tool – validate, tweak and innovate
- ✓ RAG Recommendations front page – litmus test
- ✓ RAG Tool – each recommendation – drill down
- ✓ Deepen the reading on key themes e.g. teaching for memory


Reflecting Tool

We do this and need to sustain it	We do this some of the time Do we need to maximize its potential?
We do not do this... maybe we should	What are we doing that maybe we might need to STOP doing?




IMPROVING MATHEMATICS IN KEY STAGES TWO AND THREE
A self-assessment guide

This Red Amber Green (RAG) self-assessment guide accompanies the Education Endowment Foundation's report, *Improving Mathematics In Key Stages Two and Three*, which sets out eight evidence-based recommendations on the effective teaching of mathematics.

This guide describes what 'ineffective', 'improving' and 'exemplary' practice can look like in relation to each of the recommendations.

This guide can be used as part of an initial audit process to establish current practice (i.e. point of departure), as well as to monitor progress towards the development of more effective practice (i.e. direction of travel).

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